LOGICAL DEDUCTION IN AI

PROPOSITIONAL LOGIC TO PREDICATE LOGIC

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Deduction Using Propositional Logic: Steps

Choice of Boolean Variables a, b, c, d, … which can take values true or false.

$\sim \quad \neg \qquad \vee$ (OR) $\quad \wedge$ (AND) $\quad \rightarrow \quad a \rightarrow b \quad \neg a \vee b$

Boolean Formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of Sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a Combined Formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Validity Tautology Satisfiability

Deduction Using Propositional Logic: Example 1

Choice of Boolean Variables a, b, c, d, … which can take values true or false.

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If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

$\boxed{F1: a \rightarrow b}$ $\neg a \vee b$

$\boxed{F2: a}$

$\boxed{G: b}$

$\boxed{F1 \wedge F2 \rightarrow G}$

The final formula for deduction: $(F1 \wedge F2) \rightarrow G$, that is:

$((a \xrightarrow{F1} b) \wedge \underset{}{\overset{F2}{a}}) \rightarrow \overset{G}{b}$

This formula is always TRUE Tautology

Deduction Using Propositional Logic: Example 1

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If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: $\boxed{(F1 \land F2) \rightarrow G}$, that is: $((a \rightarrow b) \land a) \rightarrow b$

TRUTH TABLES

$a \rightarrow b$ $\boxed{\neg a \vee b}$

$p \rightarrow q$ $\neg p \vee q$

a	b	a → b	(a → b) ∧ a	((a → b) ∧ a) → b
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Deduction Using Propositional Logic: Example 2

Boolean variables a, b, c, d, … which can take values true or false.

Boolean formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

If I am the President then I am well-known. I am not the President. So I am not well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: a → b

F2: ~a

G: ~b

The final formula for deduction: (F1 Λ F2) → G, that is ((a → b) Λ ~a) → ~b

F1ΛF2→G is a tautology

VALID

interpretation

T F

¬a=T

a	b	a → b	(a → b) Λ ~a	((a → b) Λ ~a) → ~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Insufficiency of Propositional Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

all some no

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

Predicate Logic

First Order Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

New Additions in Proposition (First Order Logic)

function symbols

Variables, Constants, Predicate Symbols and

New Connectors: ∃(there exists), ∀(for all)

some ∃

all ∀

arguments

goes(x, y) : x goes to y

variables

predicate symbol

b

contractor(x) dependable(x)

engineer(x)

Formulating Predicate Logic Statements

New Additions in Proposition (First Order Logic)

Variables, Constants, Predicate Symbols and

New Connectors: ∃ (there exists), ∀(for all)

Example 1!

Wherever Mary goes, so does the Lamb. Mary goes to School. So the Lamb goes to School.

Predicate: goes(x,y) to represent x goes to y

New Connectors: ∃ (there exists), ∀(for all)

F1: ∀x(goes(Mary, x) → goes(Lamb, x))

F2: goes(Mary, School)

G: goes(Lamb, School)

Mary Lamb
School

To prove: (F1 Λ F2) → G) is $\boxed{\text{always true}}$

VALID

Example 2

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

Predicates: contractor(x), dependable(x), engineer(x)

F1: ∀x(contractor(x) → ~dependable(x))

[Alternative: ~∃x (contractor(x) Λ dependable(x))]

F2: ∃x(engineer(x) Λ contractor(x))

G: ∃x(engineer(x) Λ ~dependable(x))

To prove: (F1 Λ F2) → G) is always true

$\exists x\,(\text{engineer}(x) \rightarrow \text{contractor}(x))$

$\wedge\ \exists x\ \text{engineer}(x)$

$F1 \wedge F2 \rightarrow G$

More Examples

Example 3 : —

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

graceful(x)   student(x)

dancer(x)   Ayesha

F1: $\forall x \{ \text{dancer}(x) \to \text{graceful}(x) \}$

~~$\forall x \{ \text{dancer}(x) \wedge \text{graceful}(x) \}$~~

F2: student(Ayesha)

F3: dancer(Ayesha)

G: $\exists x \{ \text{student}(x) \wedge \text{graceful}(x) \}$

$[(F1 \wedge F2 \wedge F3) \to G]$

Example: 4

Every passenger is either in first class or second class. Each passenger is in second class if and only if the passenger is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

$p(x)$, $f(x)$, $s(x)$, $w(x) \to$ wealthy

↳ passenger ↳ first class   second class

F1: $\forall x \{ p(x) \to (f(x) \vee s(x)) \}$ ✓

F1': $\forall x \{ p(x) \to \{ (f(x) \wedge \neg s(x)) \vee (\neg f(x) \wedge s(x)) \} \}$ ✓

F2: $\forall x \{ p(x) \to ((s(x) \to \neg w(x)) \wedge (\neg w(x) \to s(x))) \}$

F3: $\exists x \{ p(x) \wedge w(x) \}$

F4: $\neg (\forall x \{ p(x) \to w(x) \})$ F4: $\exists x \{ p(x) \wedge \neg w(x) \}$

G: $\exists x \{ p(x) \wedge s(x) \}$ $(F1 \wedge F2 \wedge F3 \wedge F4) \to G$

Thank you

Propositional

to Predicate Logic

variables constants

predicates

$\forall$ forall $\exists$ there exists

→ codification of sentences into formulae

→ Development of the combined formula

→ VALID / SATISFIABLE etc